



Short Range Field and Pressure - Potential Dependent Lorentz Transformation from Plasma Equation

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Abstract: Plasma equation was used in this work to derive a new nonlinear Lorentz transformation, beside, a new special relativistic energy relation dependent on potential, pressure and thermal energy. This expression reduces to the ordinary special relativity, and conforms with Newton thermodynamic and plasma energy equations. The plasma equation also predicts that photon pressure and electron gas potential can produce short range repulsive field.

Keywords: Short range field, Pressure, Potential, Lorentz transformation, Plasma equation.

1. Introduction

Plasma state is the fourth state of matter, which is an ionized fluid in which electric and magnetic field plays an important role [Yousif Albanay, 2013 and Hashim M.Ali et al 2016]. Thus fluid equations aspect more general than ordinary fluid equation, since the particles are affected by fields beside pressure effect [M.Dirar et al 2016].

This general nature of plasma equations promotes many scientists to derive new energy expressions, and quantum equation by Daniel Fleisch 2008. On the other hand, special relativity (SR) energy relation seems to be in direct conflict with plasma energy equation which cannot recognizing with some parameters such as thermal, pressure and potential energy which was done by Itzykson, C., and Zuber, J.B., (1980). This needs

to search for a new modified SR version which accounts for these defects and tries to cure them. The Lorentz transformation which is pressure and potential dependent has been done in this work. The general nature of plasma equations is also used to describe the behavior of stellar and quasi-stellar objects specially collapsing stars like black holes by Audouze, J. 1980.

2. Lorentz Pressure and Potential dependent transformation

According to plasma equation when pressure only affects the motion (De Aquino, F. 2010)

$$nm \frac{dv}{dt} = mna = \frac{-dP}{dx} \quad (1)$$

For thermal pressure:

$$PV = NkT, P = \frac{N}{V} kT \quad \text{From which}$$

$$P = nkT \quad (2)$$

Where $n = \frac{N}{V}$ number of particles per unit volume (density)

For constant uniform n

$$nma = -n \frac{dkT}{dx} \quad \text{Then}$$

$$\int madx = - \int dkT = -kT$$

For constant mass m and acceleration a

$$max = -kT \quad (3)$$

But more generally for constant acceleration and constant number density (Morse. P.M., Feshbach, H., (1953)

$$nma = \frac{-dP}{dx}$$

Then

$$\int adx = - \int \frac{dP}{nm} = - \int dP_0$$

thus

$$ax = -P_0 \quad (4)$$

$$\text{Where } dP_0 = \text{change of pressure for one particle per unit mass} \quad (5)$$

This is similar to $\nabla\phi$ which is potential change per unit mass. Consider now the effect of pressure and potential on the particle. This makes equation (1) becomes:

$$nm \frac{dv}{dt} = man = -mn \frac{dP_0}{dx} - mn \frac{d\phi}{dx} \quad (6)$$

Again when a is constant

$$ax = -P_0 - \phi \quad (7)$$

Consider now Lorentz transformation for two frames x and x' such that

$$x = \gamma(\acute{x} + L) \quad (8)$$

$$\acute{x} = \gamma(x - L) \quad (9)$$

Where L is

$$L = v_0 t + \frac{1}{2} a t^2 = v_0 t + a t^2 - \frac{1}{2} a t^2 = (v_0 + at)t - \frac{1}{2} \frac{ax}{x} t^2 = vt + \frac{1}{2} \left(\frac{P_0 + \phi}{x} \right) t^2 \quad (10)$$

$$x = \gamma \left(\acute{x} + vt + \frac{1}{2} \left(\frac{\phi + P_0}{x} \right) t^2 \right) \quad (11)$$

$$\acute{x} = \gamma \left(x - vt - \frac{1}{2} \left(\frac{\phi + P_0}{x} \right) t^2 \right) \quad (12)$$

Let

$$\phi + P_0 = D \quad (13)$$

For pulse of light moving with speed c

$$x = ct \wedge \acute{x} = c\acute{t} \quad (14)$$

Thus

$$ct = \gamma \left(c\acute{t} + vt + \frac{D}{2ct} t^2 \right) = \gamma c \left(1 + \frac{v}{c} + \frac{1}{2} \frac{D}{c^2} \right) \acute{t} \quad \text{or} \quad t = \gamma \left(1 + \frac{v}{c} + \frac{1}{2} \frac{D}{c^2} \right) \acute{t} \quad (15)$$

Similarly

$$c\acute{t} = \gamma \left(ct - vt - \frac{1}{2} \frac{D}{ct} t^2 \right) = \gamma c \left(1 - \frac{v}{c} - \frac{1}{2} \frac{D}{c^2} \right) t$$

$$\acute{t} = \gamma \left(1 - \frac{v}{c} - \frac{1}{2} \frac{D}{c^2} \right) t = \gamma(1 - f)t \quad (16)$$

Where

$$f = \frac{v}{c} + \frac{D}{2c^2} = \frac{v}{c} + \frac{(\phi + P_0)}{2c^2} \quad (17)$$

Hence substitute equation (16) into (15) yields

$$t = \gamma^2 (1 + f)(1 - f)t \quad \text{so that } 1 = \gamma^2 (1 - f^2) \quad \text{and } \gamma = (1 - f^2)^{-\frac{1}{2}}$$

Thus

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c} + \frac{\phi + P_0}{2c^2}\right)^2}} \quad (18)$$

Therefore, the energy is given by $E = mc^2 = \gamma m_0 c^2$

For small v and ϕ , such that $\phi \ll c^2 P_0 \ll c^2$

$$E = m_0 c^2 \left(1 - \left(\frac{v}{c} + \frac{(\phi + P_0)}{2c^2}\right)^2\right)^{-\frac{1}{2}} = m_0 c^2 \left[1 + \frac{1}{2} \left(\frac{v}{c} + \frac{\phi + P_0}{2c^2}\right)^2\right] \quad (19)$$

$$E = m_0 c^2 + \frac{1}{2} m_0 v^2 + \frac{v}{c} \left(\frac{\phi + P_0}{2}\right) + \frac{\phi^2}{4c^2} + \frac{P_0^2}{4c^2} + \frac{\phi P_0}{2c^2} \quad (20)$$

It is very interesting to note that the presence of pressure in the energy expression looks like the general relativity where

$$T_{\mu\nu} = g_{\mu\nu} \rho + (\rho + P) U_\mu U_\nu \text{ In the Minkowskian Limit}$$

It is also important to note that equation (19) can be simplified further by assuming

$$\frac{v}{c} < \frac{\phi + P_0}{2c^2}$$

To get

$$\left(\frac{\phi + P_0}{2c^2}\right)^2 \left(1 + \frac{2vC}{\phi + P_0}\right)^2 = \left(\frac{\phi + P_0}{2c^2}\right)^2 \left(1 + \frac{4Cv}{\phi + P_0}\right) = \left(\frac{\phi + P_0}{2c^2}\right)^2 + \frac{v}{c} (\phi + P_0) \quad (21)$$

It is clear that the energy expression consists of potential beside pressure term

3. Short range force due to photon pressure

Consider Newton second law where the force is related to the momentum, i.e. by Fatma Madani et al 2013.

$$F = \frac{dp}{dt} = \frac{d(Mv)}{dt} \quad (22)$$

For $v = \text{constant}$, (22) becomes

$$F = v \frac{dM}{dt} \quad (23)$$

Where $\frac{dM}{dt}$ is mass flow rate and can be given for constant speed flow by:

$$\frac{dM}{dt} = m \cdot n \cdot v \cdot A$$

Where n is number of photons, A cross section area and v is speed of the particle.

Insert $\frac{dM}{dt}$ in equation (23) we get:-

$$F = v. (mnvA) = mnv^2 A \quad (24)$$

For photon $v = c$ where c is speed of light, Thus

$$F = mnAv^2 = mnAc^2 = nAmC^2 \quad (25)$$

This relation can also be derived by using dimensional analysis where

$$F = \frac{mnAxv^2}{x} = \frac{mNc^2}{x} \quad (26)$$

$N = n \times A$ total numbers of photon in the included volume.

$$F = \frac{Mc^2}{x} = \frac{\text{energy}}{x} \quad (27)$$

Where

$M = mN$ is the total mass.

But the pressure is given by

$$P = \frac{F}{A} = mnc^2 = 2 \frac{S_{avg}}{c} \quad (28)$$

Where S_{avg} is energy flow per unit time per unit area, also

$$S_{avg} = \frac{1}{2} (mc^2 n) c \quad (29)$$

Where $mc^2 n$ represents energy density

$$I = \text{intensity} = (\text{energy density}) \times c \quad (30)$$

$$F = \frac{\partial V}{\partial x} \frac{\text{Energy}}{\text{distance}} \quad (31)$$

Consider a photon as a particle moving inside fluid of photons in gravity field of potential per unit mass ϕ . Assume also the photon is affected by fluid force. thus

$$F = mnc^2 A \quad (32)$$

Thus the photon equations is

$$mn \frac{dv}{dt} = F - m \nabla \phi \quad (33)$$

Since the photon moves with constant speed thus

$$mn \frac{dv}{dt} = mn \frac{dc}{dt} = 0 \quad (34)$$

Thus the equation of motion of photon is given by

$$m \nabla \phi = m \frac{d\phi}{dx} = F = mnAc^2 \quad (35)$$

Considering the photon as gas obeying Maxwell-Boltzmann distribution

$$n = n_0 e^{-\beta \phi} \quad (36)$$

For very small value of β such that

$$\beta \phi < 1$$

$$n = n_0 (1 - \beta \phi) \quad (37)$$

Inserting (33) in (32) yields

$$m \frac{d\phi}{dx} = m n_0 A C^2 (1 - \beta \phi) = C_1 f$$

$$\frac{d\phi}{dx} = n_0 A C^2 (1 - \beta \phi) = C_1 f \quad (38)$$

Where

$$C_1 = n_0 A C^2$$

$$f = 1 - \beta \phi \quad (39)$$

Thus

$$\frac{df}{dx} = -\beta \frac{d\phi}{dx} \quad (40)$$

Hence equation (39) yields

$$\frac{-1}{\beta} \frac{df}{dx} = C_1 f$$

Rearranging gives

$$\int \frac{df}{f} = -C_1 \beta \int dx + C_2, \ln f = -C_1 \beta x + C_2$$

$$f = e^{C_2} e^{-C_1 \beta x} = C_3 e^{-C_1 \beta x} \quad (41)$$

From equation (39)

$$\phi = \frac{1}{\beta} (1 - C_3 e^{-C_1 \beta x}) \quad (42)$$

$$\text{at : } x=0, \phi = \phi_0$$

$$\text{Hence: } \phi_0 = \frac{1}{\beta} (1 - C_3)$$

$$\text{At: } x \rightarrow \infty$$

$$\phi = \phi_c = \text{cosmic potential}$$

Thus

$$\phi_c = \frac{1}{\beta}$$

Thus

$$\phi = \frac{1}{\beta} [1 - (1 - \beta\phi_0)e^{-c_1\beta x}]$$

$$\phi = [\phi_c + (1 - \phi_c)e^{-c_1\beta x}] \quad (43)$$

4. Short range field generated by electron gas in crystals

Consider an electron surrounded by electron cloud of density n and each contributes an electric field E_0 the force on the electron (Matthew N.O.Sadiku, 2001) is:

$$m \frac{dv}{dt} = -\nabla\phi + nE_0 = -\nabla\phi - E_0 n_0 e^{-\beta\phi} \quad (44)$$

$$\text{Let } c_0 = n_0 E_0$$

Using Taylor's series $e^{-\beta\phi}$ can be expanded in the form.

$$e^{-\beta\phi} = 1 - \beta\phi + \frac{(\beta\phi)^2}{2!} + \frac{(\beta\phi)^3}{3!} + \dots \cong 1 - \beta\phi \quad (45)$$

For small β , such that $\beta\phi < 1$, $e^{-\beta\phi} \approx 1 - \beta\phi$

Thus equation (33)

$$m \frac{dv}{dt} = -\nabla\phi + c_0(1 - \beta\phi) \quad (46)$$

Consider uniform motion, such that

$$\frac{dv}{dt} = 0$$

By considering that the electron speed is to be uniform. Hence equation (46) reduced to

$$\nabla\phi = c_0(1 - \beta\phi) \quad (47)$$

Or

$$\nabla\phi = c_0(1 - \beta\phi) \quad (48)$$

$$\frac{d\phi}{dx} = c_0(1 - \beta\phi) = c_0 - c_0\beta\phi \quad (49)$$

Define f such that

$$\frac{d\phi}{dx} = c_0 - c_0\beta\phi = f \quad (50)$$

Now let

$$c_1 = c_0\beta$$

Thus

$$c_0 - c_1\phi = f \quad (51)$$

Differentiating equation (51) yields

$$-c_1 d\phi = df$$

$$d\phi = \frac{-1}{c_1} df \quad (52)$$

Thus equation (50) gives

$$\frac{-1}{c_1} df = f dx \quad (53)$$

$$\frac{df}{f} = -c_1 dx \quad (54)$$

Integrating both side yields

$$\ln f = -c_1 x + c_2 \quad (55)$$

Where c_2 is constant of the integration.

$$f = e^{-c_1 x + c_2} \quad (56. a)$$

$$f = ce^{-c_1 x} \quad (56. b)$$

Where

$$c = e^{c_2}$$

Equate equation (50 and 56) to get

$$c_0 - c_1\phi = ce^{-c_1 x} \quad (57)$$

Let

$$c_3 = \frac{c_0}{c_1}, c_4 = \frac{c}{c_1}$$

$$\phi(x) = c_3 - c_4 e^{-c_1 x} \quad (58)$$

Since self-energy exists the origin, hence

$$\text{At } x = 0 \quad \phi(0) = c_3 - c_4 = \phi_0$$

Also vacuum energy exists at infinity hence

$$\text{And } x = \infty \quad \phi(\infty) = c_3 = \phi_v$$

Therefore

$$\phi(0) = \phi_0 = \phi_v - c_4 \quad (59)$$

Hence

$$C_4 = \phi_v - \phi_0$$

Inserting equation (58 in to 59) yields

$$\phi = \phi_v + (\phi_0 - \phi_v)e^{-C_2x} \quad (60)$$

For negligible vacuum energy

$$\phi = \phi_0 e^{-C_2x} \quad (61)$$

5. Discussion

The SR energy relation appears to be in conflict with plasma and thermodynamics energy equations (21 and 43) which consists of thermal beside mechanical pressure energy, where

$$E_{sr} = m_0 \left(1 - \frac{v^2}{c^2}\right)^{\frac{-1}{2}}$$

$$E_{plas} = \frac{1}{2}m_0v^2 + V + \gamma kT + P_m$$

$$dQ = dU + PdV = dU + dw$$

This conflict is removed here by suggesting new Lorentz transformation, nonlinear in time see equations (11) and (12). By assuming homogeneity of space - time and equivalence of coordinates, beside constancy of speed of light in vacuum, one finds new Lorentz transformation which reduces to the ordinary one when no potential or pressure exists see equation (23) . It is also reduces to ordinary plasma energy.

The stability of stars against collapse to avoid singularity is one of the important issues in cosmology and particle physics. Section (2) proves that photon pressure inside black holes or any collapsing stellar object can support collapse by generating short range repulsive gravitational or nuclear field as shown by equation (43) Here one uses plasma equation (36) by considering the photon number density a gas obeying Maxwell equation (which is a limiting case of Bose – Einstein distribution when $e^{\beta\phi} > 1$). Then by expanding the exponential, potential term, as a Taylor series, one gets a useful equation see equation (44) which leads to short range potential. For negligible cosmic field

$$\phi = e^{-C_1\beta x}$$

which is a short range repulsive field. Similarly electron gas within crystal can also generate short range field due to the effect of the crystal and electric field [see equation (44)], assuming electron density obeys Maxwell distribution [see equation (36)] and β to b small, one can find a useful equations (46 and 50) for uniform velocity. These expressions predicts the existence of short range field [see equation (61) affected by self-energy and vacuum energy for negligible vacuum energy equation (56) shows presence of short range repulsive field.

6. Conclusion

Plasma equation general nature make it capable of recognizing the effect of potential, pressure and thermal energy in SR. this enables generalizing Lorentz transformation and special relativity to recognize the effect of potential and pressure effects. Thus makes SR to conform to plasma equations and thermodynamics. It also helps in deriving short range repulsive energy. This opened the door for considering cosmological model which is free of singularity. It also raises a hope in describing black holes by nonsingular model which prevents gravitational collapse.

Using Newton Poisson equation beside Maxwell statistical distribution law one can predict existence of short range repulsive gravity force. This gravity force may have a link with strong nuclear force and predicts finite self-mass energy and help in constructing nonsingular model for stars and universe. It also enables using describing nonsingular and non-collapsing black holes. By using Newton's second law for constant acceleration one can drive new Lorentz transformation which recognizes the effect of potential as well as velocity. This reduces to that SR besides predicting the gravity time dilation.

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