Fundamental Parameters Affecting Non Linearity of Suspension Bridges

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Abstract

In this research factors affecting the non-linearity of suspension bridges were studied. The fundamental parameters studied are the main and side span lengths, cable sag, tower height, cable x-section and the flexural rigidity of the stiffening girder .The effect of variation of each parameter on the cable tension and the girder moments is studied.

A non-linear 2-dimensional mathematical model of a 3 span, continuous suspension bridge is considered . The solution is based on the second order deflection theory given in a computerized form. It has been found that the degree of effect of these parameters on the results of analysis in a descending order is: the main span length, side span length, cable sag, cable section and the stiffness of the bridge girder.

ملخص

في هذا البحث تم دراسة اثر العوامل المؤثرة على لا خطية الجسور المعلقة، والعوامل الرئيسةهى :طول البحر الرئيس والبحر الجانبى وارتخاء الكيبل وارتفاع البرج ومقطع الكيبل وجساءة الانحناء لعارضة التقوية. وقد تم دراسة اثر تغيير كل عامل على مقدار الشد فى الكيبل والعزوم فى عارضة التقوية.

تم اعتبار نموذج رياضى لاخطى لجسر معلق مستمر ثنائى الابعاد ثلاثى البحور ، وقد حل هذا النموذج على اساس نظرية الانحراف اللاخطية والتى برمجت على الحاسب .ولقد وجد ان درجة تاثير العوامل اعلاه عاى لاخطية التحليل تؤثر تنازليا كما يلى: طول البحر الرئيس ثم البحر الجانبى ثم ارتخاء الكيبل ثم مقطع الكيبل ثم صلابة عارضة التقوية.

1. Introduction

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Cable structures (1), have been known to undergo large deformations due to their flexibility. This is termed geometric non-linearity which require special analysis. Among those cable structures, externally-anchored Suspension Bridges are mostly affected by non-linearity. The major suspension bridge elements are shown in Fig. (1).

Historically (2), suspension bridges analysis passed through several developments. The main theories and their features are as follows :

(A) Rankine theory: An old theory originated by Rankine in 1885 .The theory assumed that the applied live load on the bridge deck will be uniformly resisted by equal hangers tension. Of course this is an approximate theory yielding inaccurate results .

(B) The Elastic theory: considers the cable as an inverted arch . It is common and approximate and yields too high values of bending moments and shears in the girder . The theory does not consider the effect of change of cable geometry.

(C) The Deflection theory: a more exact method, which takes account of deformed configuration of the structure. The theory was originated by Melan in 1888, extended by Moisseff and Steinman for continuous suspension bridges; and then generalized by Steiman in 1935 (3,4). The theory can be summarized as follows.

2. The Deflection Theory (2,3,4,5,6,7,8)

a-General

Analysis of suspension bridges by the deflection theory is based on nonlinear formulation and provides appropriate provision for the influence of deflections in changing the geometry. The theory is based on the differential equation of the cable and the stiffening girder, together with the cable equation, which governs the elastic extension of the cable and the shortening arising from the deflection along the cable.

b- Basic Assumptions

(1) The initial curve of the cable is a parabola.

(2) The initial dead load, w, is carried by the cable only, producing the initial horizontal tension, H.

(3) Suspenders elongations are negligible

(4 The deflection of any point on the cable, v(x), due to applied live load "p" per unit length of the girder, is sensible .

c- Fundamental Equations

The bending moment at any point of a simple suspended span is given by (10):

$$M = M_0 - hy \qquad (1)$$

Where h is the induced tension due to live load.

. For amount of deflection v, the bending moment will be reduced to

$$M = [M_0 - hy] - (H + h)\, v \qquad (2)$$

For a continuous suspended middle span, accounting for continuity, equation (2) will take the form

$$M = [M_0 - hy] - (H + h)\, v + T \qquad (3)$$

where T, is the continuity moment and is given by

$$\mathrm{T} = \mathrm{T}_1 - \frac{(\mathrm{T}_1 + \mathrm{T}_2)}{\mathrm{L}}\, \mathrm{x} \qquad (4)$$

It can been proven that (10)

$$H = \frac{wL^2}{8d} \qquad (5)$$

Denoting

$$\mathrm{C}^2 = \frac{\mathrm{H} + \mathrm{h}}{\mathrm{EI}} \qquad (6)$$

and, since $M = -EIv''$, equation (3) becomes

$$v'' = C^2 v - \frac{C^2}{H + h}(M_0 + T - hy) \qquad (7)$$

Equation (7) is the first fundamental equation; known as the girder equation. There are two unknowns; namely the deflection v and the induced tension "h ". The other fundamental equation, known as the cable equation is (2,3,9):

$$\frac{hl}{A_c E_c} \pm \propto L_c t - \frac{w}{H}\int_0^L v\,dx = 0 \qquad (8)$$

in which

$$l = \int_0^L \left(\frac{ds}{dx}\right)^3 dx \approx L\left(\sec^3\theta + 8\frac{d^2}{L^2}\sec\theta\right) \qquad (9)$$

and

$$L_c = \int_0^L \left(\frac{ds}{dx}\right)^2 dx \approx L\left(\sec^2\theta + \frac{16}{3}\frac{d^2}{L^2}\right) \qquad (10)$$

The solution of equation (7) will yield the general formula for girder deflection

$$v = \frac{h}{h+H}\left[C_1 e^{Cx} + C_2 e^{-Cx} + \left(\frac{M_0}{h} + \frac{T}{h} - y\right) + \frac{1}{C^2}\left(\frac{8d}{L^2} - \frac{p}{h}\right)\right] \qquad (11)$$

where C_1 andC_2 are integration constants.

d- Solution of Analytical Model by the Deflection Theory

In this section, the general 2-dimensional, 3-span, continuous suspension bridge model, shown in Fig. (2) is solved, on the basis of the deflection theory. The application of equation, (7), to any span of the model necessitates the evaluation of the followings:

i) Three pairs of the integration constants C_1 and C_2 for the three span segments k, m, and j; and this is done by applying boundary conditions.

ii) The girder continuity moments T_1 and ($-T_2$)at the towers; the continuity of the slopes at the towers is used here.

iii) The horizontal component of the induced tension (h) due to live loads; the cable equation is used here.

It will be shown hereafter that the above parameters (which are twenty one parameters for the entire bridge) are interdependent, i.e. every set of the parameters is a function of the other two sets. To overcome such complexity, an iterative computerized analysis is developed as follows.

e- Determination of Moments, Shears, Deflection & Suspenders' Tension

Referring to equation (11), the girder deflection at any point x on any span i, of length L_i, is given by:-

$$v_i(x) = \frac{h}{h+H}\left[C_1 e^{C_i x} + C_2 e^{-C_i x} + \left(\frac{M_{0i}}{h} + \frac{T}{h} - y_i\right) - \frac{1}{C_i^2}(\frac{p_x}{h} - \frac{8d_i}{L_i^2}\right] \qquad (12)$$

Differentiating eq. (12) with respect to x will give the slope of the girder:

$$v_i'(x)=\frac{C_i h}{h+H}\left[C_1 e^{C_i x}-C_2 e^{-C_i x}+\frac{1}{C_i}\left(\frac{M_{0i}'}{h}+\frac{T'}{h}-y_i'\right)\right] \tag{13}$$

But $M=-EIv''$, and so the moment at any section x is

$$M_i(x)=-h\left[C_1 e^{C_i x}+C_2 e^{-C_i x}+\frac{1}{C_i^2}\left(\frac{8d_i}{L_i^2}-\frac{p_x}{h}\right)\right] \tag{14}$$

The shear is $V=\dfrac{dM}{dx}$

Substituting

$$V_i(x)=-C_i h\left[C_1 e^{C_i x}-C_2 e^{-C_i x}\right] \tag{15}$$

The portion of the live load carried by the suspenders per unit length, q(x), is the difference between the live load at the section p_x and the net load carried by the girder, i.e.

$$q(x)=p_x-\left(-\frac{dV}{dx}\right)=p_x-C_i^2 h\left[C_1 e^{C_i x}+C_2 e^{-C_i x}\right] \tag{16}$$

f- Computer Implementation of The solution Based on the Deflection Theory(10) :

The foregoing analysis based on the deflection theory is implemented in computer form here. An iterative algorithm to determine the value of h satisfying all equation is followed. The FORTRAN computer program written is named as SASDEF (static Analysis of suspension bridges using the Deflection theory). Two subroutines serve the main program:

(1) SUBROUTINE CIC2: for calculation of the integration constants sets C_1, C_2.

(2) SUBROUTINE CONT: for calculation of the continuity moments of the girder at the towers.

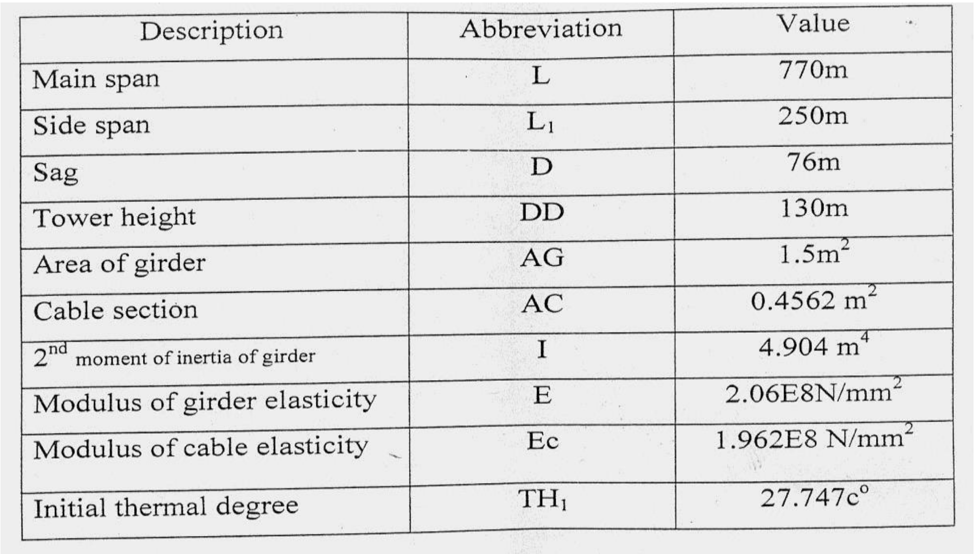
Few entries are needed, and the program reveals a complete solution to the girder- cable interaction of the bridge ,see flow chart, Fig.(3).

g- Effect of Fundamental Parameters

Data Information of an Existing Suspension Bridge:

In this chapter using the computer program for the second order theory for a real data bridge (Innoshima, Japan at 1983, with m3ain span 770 m) (11), the parameter information data of this bridge can be organized as below:

Table (1): The Information Data of Innoshima Bridge



Description	Abbreviation	Value
Main span	L	770m
Side span	L_1	250m
Sag	D	76m
Tower height	DD	130m
Area of girder	AG	$1.5m^2$
Cable section	AC	$0.4562\ m^2$
2^{nd} moment of inertia of girder	I	$4.904\ m^4$
Modulus of girder elasticity	E	$2.06E8N/mm^2$
Modulus of cable elasticity	Ec	$1.962E8\ N/mm^2$
Initial thermal degree	TH_1	$27.747c^o$

Table (2) Live Load Data of Innoshima Bridge (assumed):

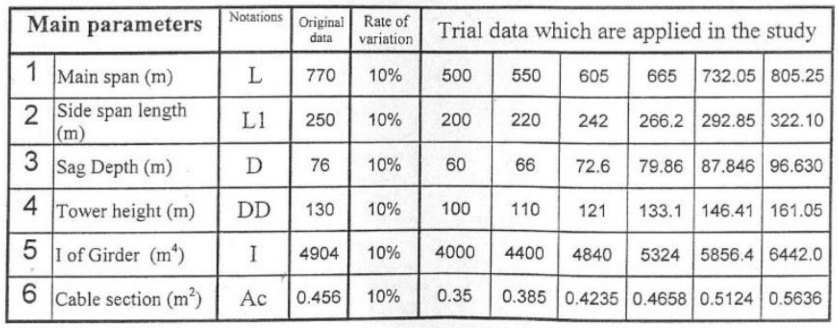
Description	Abbreviation	Value
Load Rite side span	P	29.6kN/m

Segment no .1 Right side	S1=J1/L1	1.0
Segment no .2 Right side	T1=K1/L1	0.0
Load at main span	P1	29.6kN/m
Spacing no.1 at left main span	S2=J2/L	1.0
Spacing no.2 at left main span	T2=K2/L1	0.0
Load at left side span	P2	29.6kN/m
Spacing no.1 at left main span	S3=J3/L	1.0
Spacing no.2 at left main span	T3=K3/L1	0.0

3. Trial Data

Using a fixed variation for each parameter by 10% the trial data assumed in the study is as shown Table (3), It can be recognized that the original data lie with in the trail data. The objective of these variations is to observe their influence on the non – linearity of the analysis performed by the deflection theory.

Table (3)



Main parameters		Notations	Original data	Rate of variation	Trial data which are applied in the study					
1	Main span (m)	L	770	10%	500	550	605	665	732.05	805.25
2	Side span length (m)	L1	250	10%	200	220	242	266.2	292.85	322.10
3	Sag Depth (m)	D	76	10%	60	66	72.6	79.86	87.846	96.630
4	Tower height (m)	DD	130	10%	100	110	121	133.1	146.41	161.05
5	I of Girder (m^4)	I	4904	10%	4000	4400	4840	5324	5856.4	6442.0
6	Cable section (m^2)	Ac	0.456	10%	0.35	0.385	0.4235	0.4658	0.5124	0.5636

On selecting a trial value of any parameter, we run the program to obtain the results for that parameter in-terms of:

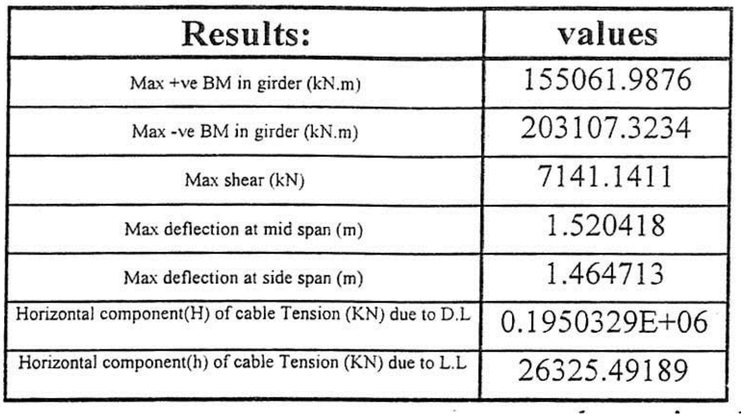
1. Max +ve BM in girder (kN.m).
2. Max-ve BM in girder (kN.m).
3. Max shear (kN).
4. Max deflection at mid span ,(m).

5. Max deflection at site span ,(m).
6. Horizontal component (H) of the cable Tension due to D.L, (kN).
7. Horizontal component (h) of the cable Tension due to L.L, (kN).

4. Results

Analysis of the results of original dimension of the bridge is illustrated in Table (4).

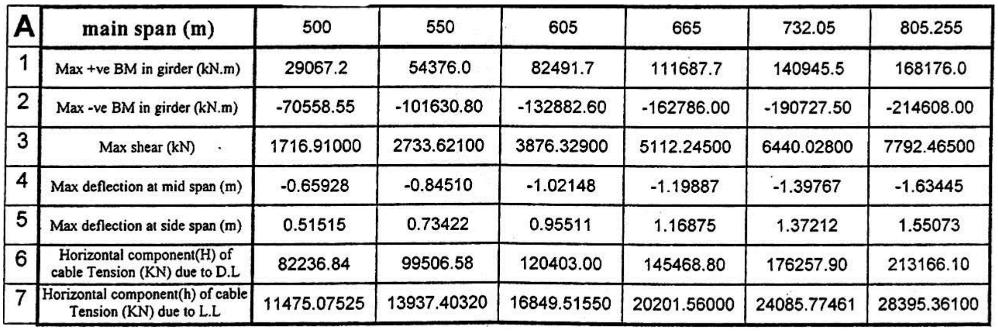
Table (4)



Results:	values
Max +ve BM in girder (kN.m)	155061.9876
Max -ve BM in girder (kN.m)	203107.3234
Max shear (kN)	7141.1411
Max deflection at mid span (m)	1.520418
Max deflection at side span (m)	1.464713
Horizontal component(H) of cable Tension (KN) due to D.L	0.1950329E+06
Horizontal component(h) of cable Tension (KN) due to L.L	26325.49189

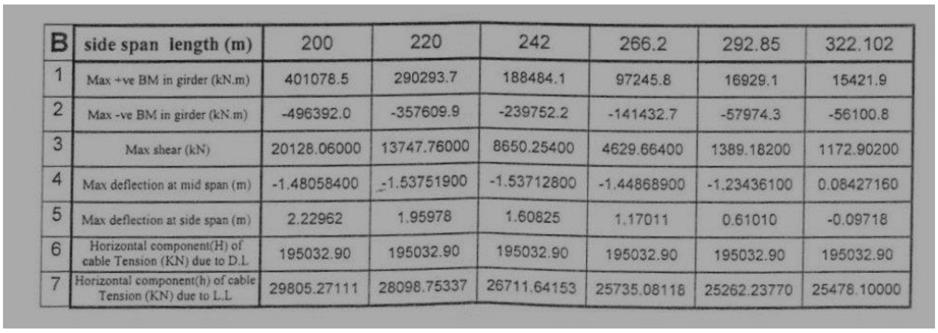
Analysis results for each parameter variation are shown in Table (5) To Table (9).

Table (5) Effect of main span length (L) on the Non–linear analysis (With constant side span length)



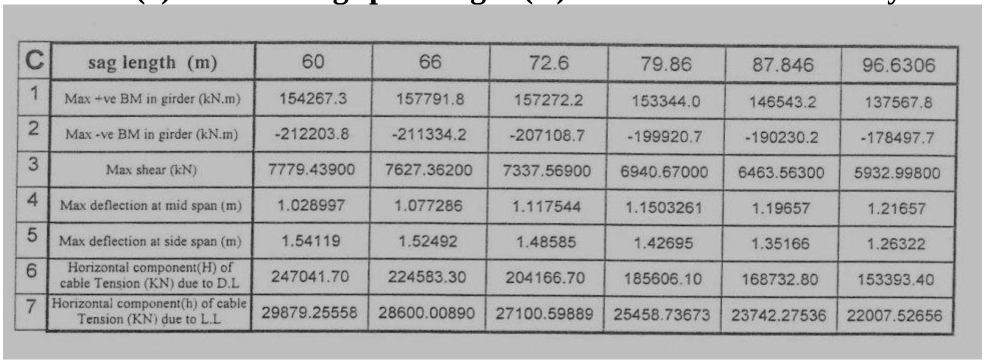
A	main span (m)	500	550	605	665	732.05	805.255
1	Max +ve BM in girder (kN.m)	29067.2	54376.0	82491.7	111687.7	140945.5	168176.0
2	Max -ve BM in girder (kN.m)	-70558.55	-101630.80	-132882.60	-162786.00	-190727.50	-214608.00
3	Max shear (kN)	1716.91000	2733.62100	3876.32900	5112.24500	6440.02800	7792.46500
4	Max deflection at mid span (m)	-0.65928	-0.84510	-1.02148	-1.19887	-1.39767	-1.63445
5	Max deflection at side span (m)	0.51515	0.73422	0.95511	1.16875	1.37212	1.55073
6	Horizontal component(H) of cable Tension (KN) due to D.L	82236.84	99506.58	120403.00	145468.80	176257.90	213166.10
7	Horizontal component(h) of cable Tension (KN) due to L.L	11475.07525	13937.40320	16849.51550	20201.56000	24085.77461	28395.36100

Table (6) Effect of side span length (L1) on the Non–linear analysis
(With constant side span length)



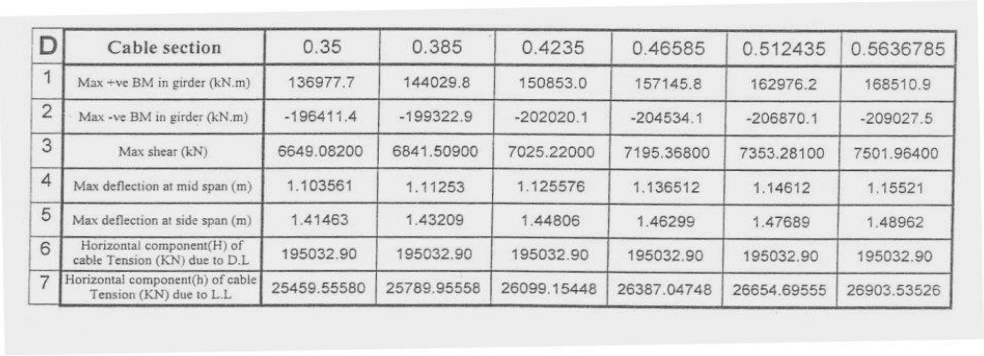
B	side span length (m)	200	220	242	266.2	292.85	322.102
1	Max +ve BM in girder (kN.m)	401078.5	290293.7	188484.1	97245.8	16929.1	15421.9
2	Max -ve BM in girder (kN.m)	-496392.0	-357609.9	-239752.2	-141432.7	-57974.3	-56100.8
3	Max shear (kN)	20128.06000	13747.76000	8650.25400	4629.66400	1389.18200	1172.90200
4	Max deflection at mid span (m)	-1.48058400	-1.53751900	-1.53712800	-1.44868900	-1.23436100	0.08427160
5	Max deflection at side span (m)	2.22962	1.95978	1.60825	1.17011	0.61010	-0.09718
6	Horizontal component(H) of cable Tension (KN) due to D.L	195032.90	195032.90	195032.90	195032.90	195032.90	195032.90
7	Horizontal component(h) of cable Tension (KN) due to L.L	29805.27111	28098.75337	26711.64153	25735.08118	25262.23770	25478.10000

Table (7) Effect of sag span length (D) on the Non–linear analysis



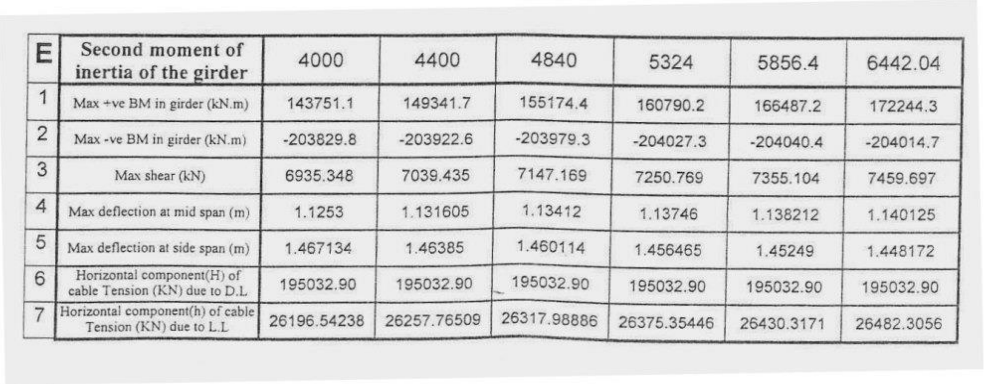
C	sag length (m)	60	66	72.6	79.86	87.846	96.6306
1	Max +ve BM in girder (kN.m)	154267.3	157791.8	157272.2	153344.0	146543.2	137567.8
2	Max -ve BM in girder (kN.m)	-212203.8	-211334.2	-207108.7	-199920.7	-190230.2	-178497.7
3	Max shear (kN)	7779.43900	7627.36200	7337.56900	6940.67000	6463.56300	5932.99800
4	Max deflection at mid span (m)	1.028997	1.077286	1.117544	1.1503261	1.19657	1.21657
5	Max deflection at side span (m)	1.54119	1.52492	1.48585	1.42695	1.35166	1.26322
6	Horizontal component(H) of cable Tension (KN) due to D.L	247041.70	224583.30	204166.70	185606.10	168732.80	153393.40
7	Horizontal component(h) of cable Tension (KN) due to L.L	29879.25558	28600.00890	27100.59889	25458.73673	23742.27536	22007.52656

Table (8): Effect of Cable x-section on the analysis



D	Cable section	0.35	0.385	0.4235	0.46585	0.512435	0.5636785
1	Max +ve BM in girder (kN.m)	136977.7	144029.8	150853.0	157145.8	162976.2	168510.9
2	Max -ve BM in girder (kN.m)	-196411.4	-199322.9	-202020.1	-204534.1	-206870.1	-209027.5
3	Max shear (kN)	6649.08200	6841.50900	7025.22000	7195.36800	7353.28100	7501.96400
4	Max deflection at mid span (m)	1.103561	1.11253	1.125576	1.136512	1.14612	1.15521
5	Max deflection at side span (m)	1.41463	1.43209	1.44806	1.46299	1.47689	1.48962
6	Horizontal component(H) of cable Tension (KN) due to D.L	195032.90	195032.90	195032.90	195032.90	195032.90	195032.90
7	Horizontal component(h) of cable Tension (KN) due to L.L	25459.55580	25789.95558	26099.15448	26387.04748	26654.69555	26903.53526

Table (9) : Effect of girder rigidity on the analysis



E	Second moment of inertia of the girder	4000	4400	4840	5324	5856.4	6442.04
1	Max +ve BM in girder (kN.m)	143751.1	149341.7	155174.4	160790.2	166487.2	172244.3
2	Max -ve BM in girder (kN.m)	-203829.8	-203922.6	-203979.3	-204027.3	-204040.4	-204014.7
3	Max shear (kN)	6935.348	7039.435	7147.169	7250.769	7355.104	7459.697
4	Max deflection at mid span (m)	1.1253	1.131605	1.13412	1.13746	1.138212	1.140125
5	Max deflection at side span (m)	1.467134	1.46385	1.460114	1.456465	1.45249	1.448172
6	Horizontal component(H) of cable Tension (KN) due to D.L	195032.90	195032.90	195032.90	195032.90	195032.90	195032.90
7	Horizontal component(h) of cable Tension (KN) due to L.L	26196.54238	26257.76509	26317.98886	26375.35446	26430.3171	26482.3056

5. Plotting the Relation between Each Parameter and its Effects on the Analysis

The preceding results (Table 5 to Table 9) are illustrated in graphical from- as shown in appendix (I), so as to observe the relation between each parameter variations and the corresponding effect on the main result analysis, namely; Max. . +ve BM in girder, Max –ve BM in girder, Max. shear in the girder , Max. deflection at mid span , Max. deflection at side span and the horizontal component (H) of cable Tension . Every chart equation is written to its best fit with regression type of polynomial trend.

6. Conclusions

The foregoing research explains how the non-linearity based on the computerized treatment of the deflection theory affects the analysis of suspension bridges .The effect of several parameters on the results analysis is studied by varying each parameter and observing the result variation .The following conclusions are derived .

- **a.** Increasing the main span length causes the rate of change of several results of analysis to decrease, namely: maximum positive bending moment in the main span girder, maximum negative bending moment in the girder, maximum shear in the girder, maximum deflection in the mid span girder and max deflection at side span girder.
- **b.** Increasing the main span length will increase the rate of increase of cable tension .This result is very important-design wise –because it proves that as the main span length increase, more load is being carried by the cable. Consequently suspenders tensions becomes now significant.
- **c.** Increasing the side span length generally will increase the rate of decrease of some results analysis namely: maximum positive bending moment in the girder , maximum negative bending moment in the girder , maximum shear in the girder ,maximum deflection at the mid span girder and max deflection at side span girder. It can be noticed that as the side span length approaches or exceeds half the value of the main span, the rate of change vanishes.
- **d.** Increasing of the side span length will cause the cable tension to decrease. This relation is valid just before the side span length approaches half the main span length, after witch the cable tension will increase.
- **e.** The deformation findings prove that optimum design length of the side span is a value of approximately half the main span length.
- **f.** The effect of sag on the analysis results is less than of the main span and side spans lengths.
- **g.** Effects of cable cross-section and the moment of inertia of the stiffening girder are not significant as shown on graphs .

h. In general it can be concluded that as the externally-anchored suspension bridges spans increase, the whole structure becomes stiffer .And the cable system will carry more addition loads.

7. References

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8. Appendices

a-Appendix (I)- Figures

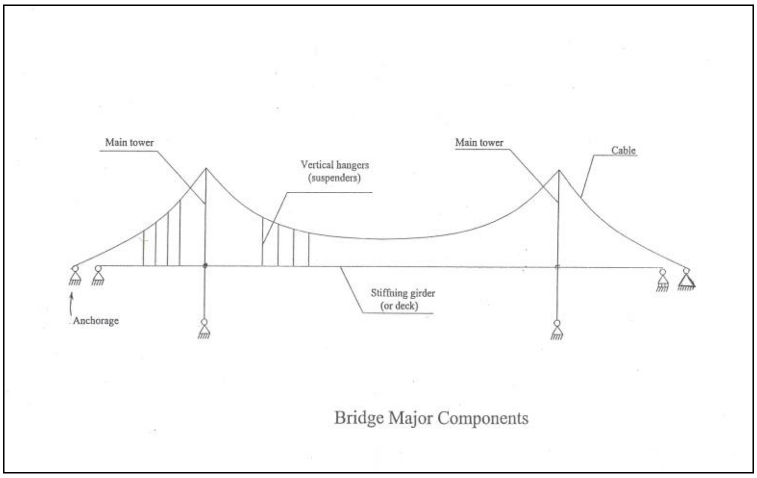


Fig (1) Bridge Major Components

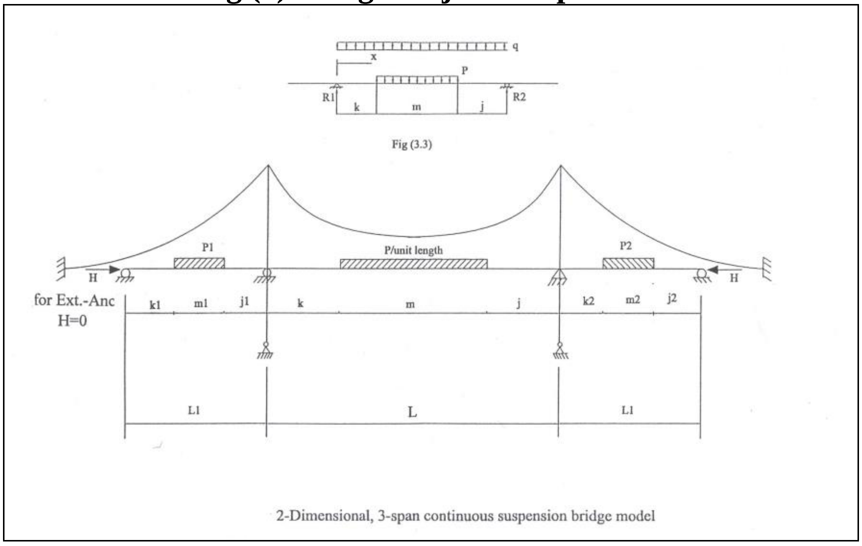
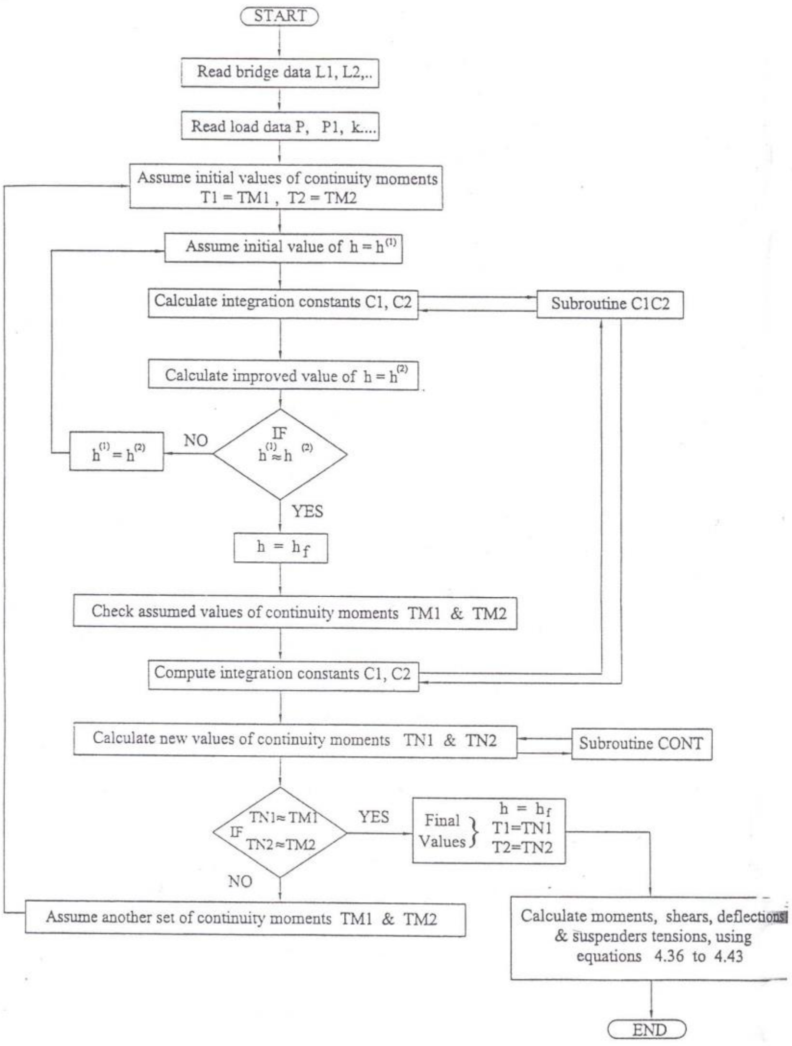


Fig (2): 2-Dimensional,3-Span Suspension Bridge Model



FlowChart - the deflection theory, Program SASDEF.FOR

Fig (3):- Flow Chart-Program (SADDEF)

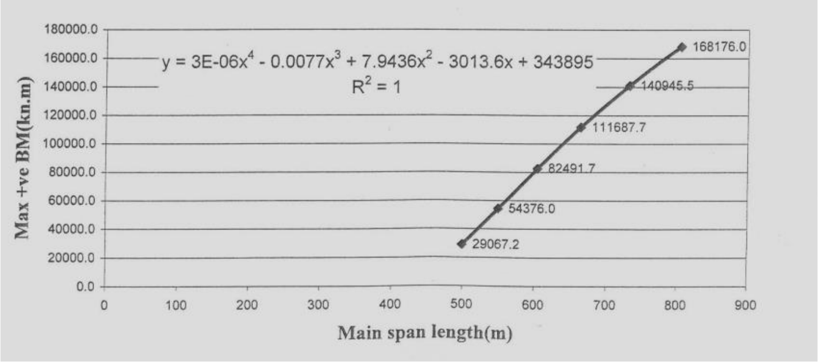


Fig (4):Max. +ve BM (kN.m) vs Main span Length,m

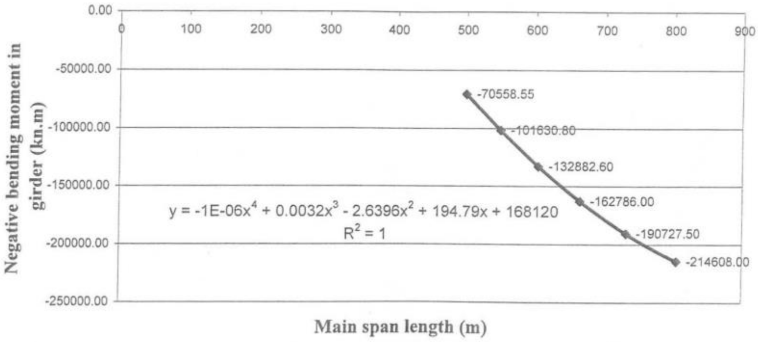


Fig (5): -ve BM (kN.m) vs Main span Length (m)

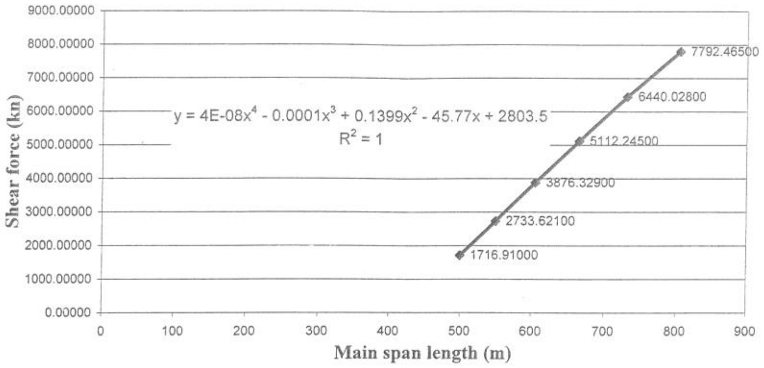


Fig (6): Shear (kN) vs Main span Length

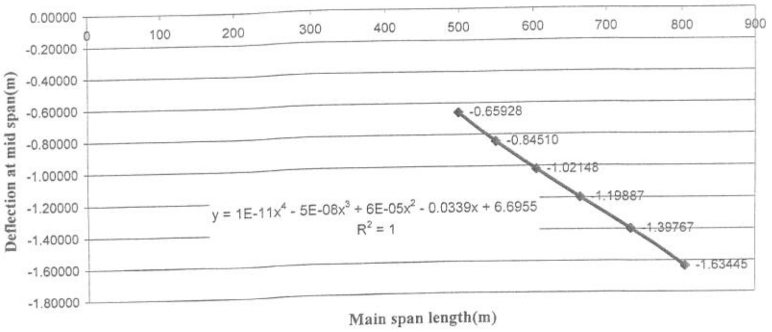


Fig (7):Max mid span deflection (m) vs. Main span Length ,m

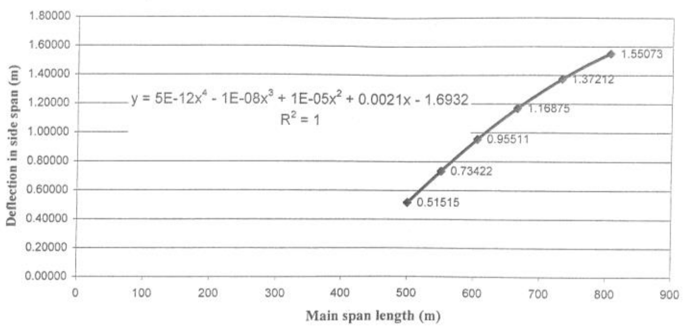


Fig (8): max Deflection in side Span vs. Main span Length

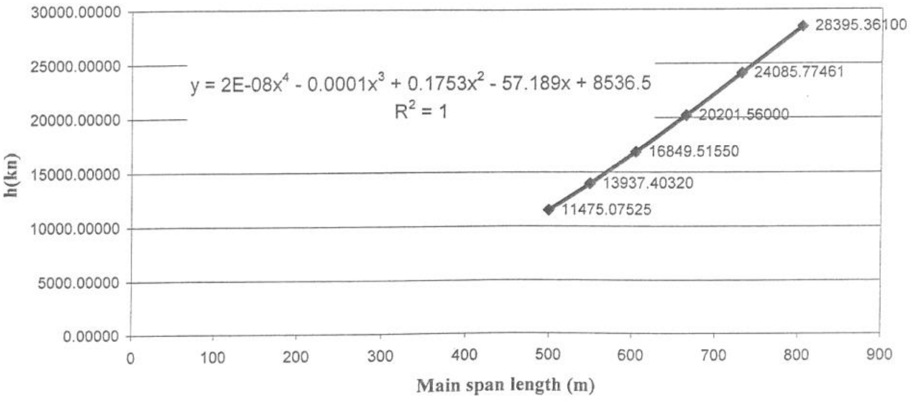


Fig (9): L.L Cable Horiz. Tension (kN) vs. Main span (m)

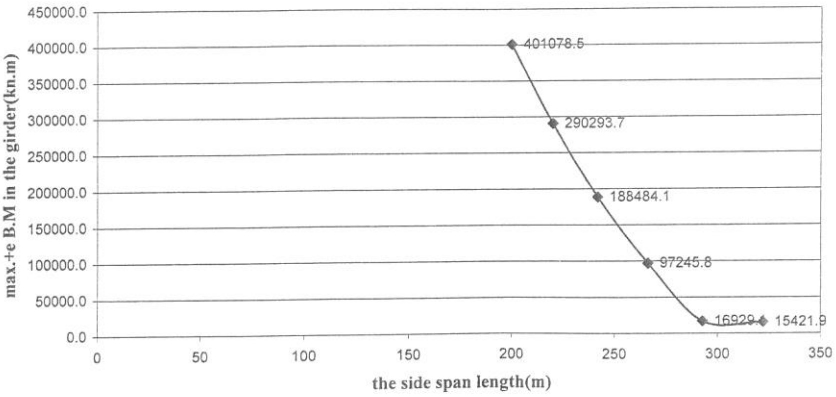


Fig (10):Max +ve BM (kN.m) vs Side span Length (m)

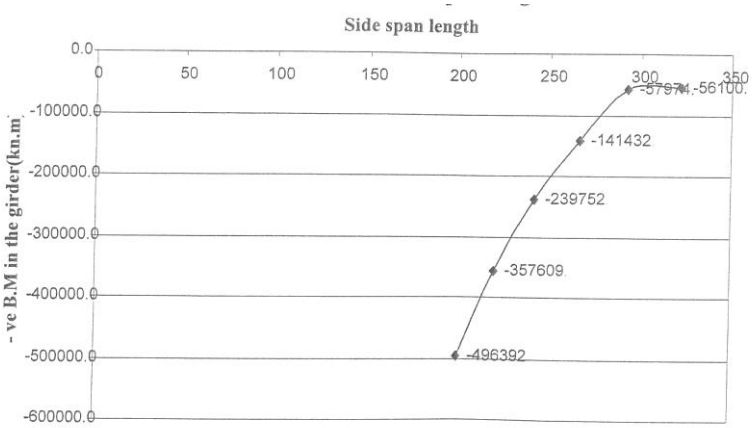


Fig (11):-ve BM (kN.m) vs. Side span Length (m)

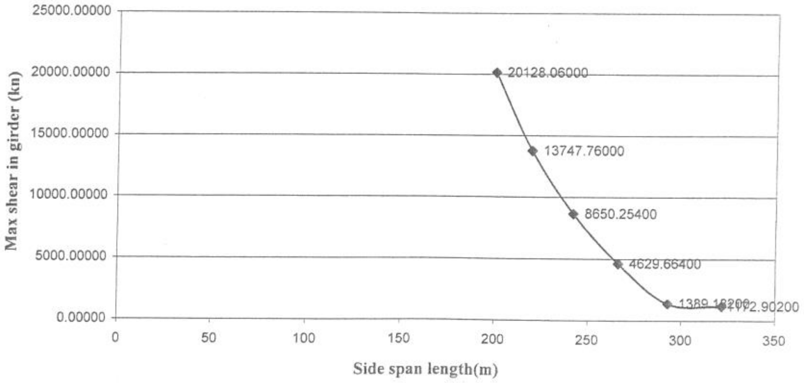


Fig (12):Max girder Shear (kN) vs. Side span length(m)

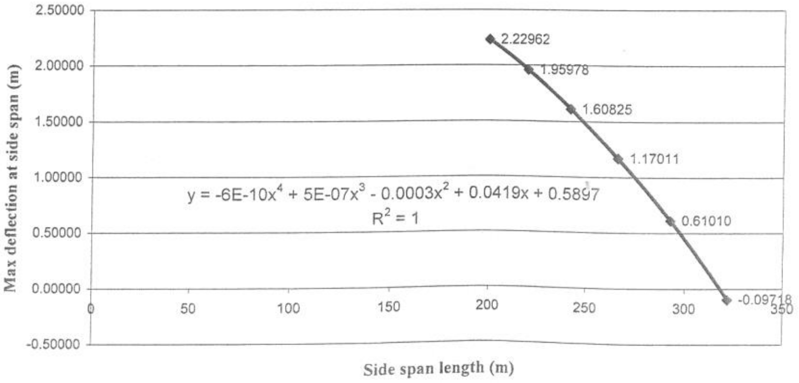


Fig (13):Max side span Deflection (m) vs. side span Length (m)

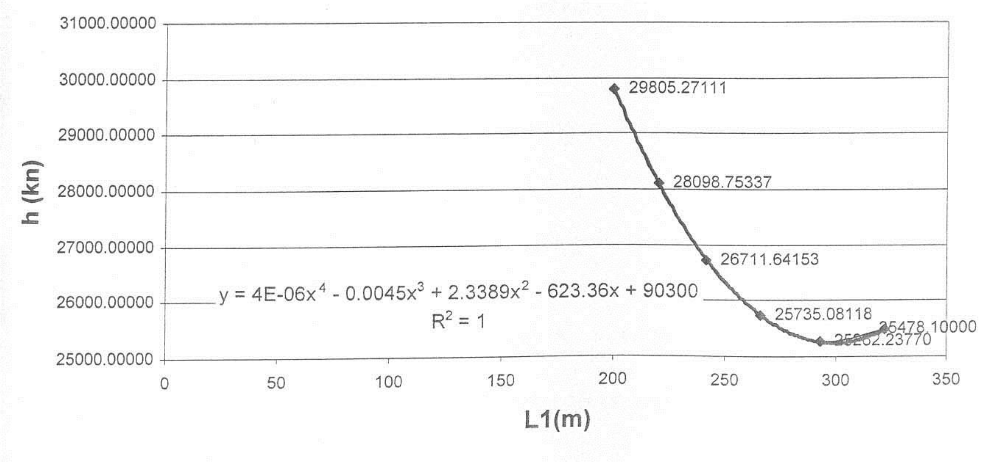


Fig (14): L.L Cable Horiz. Tension (kN) vs. side span (m)

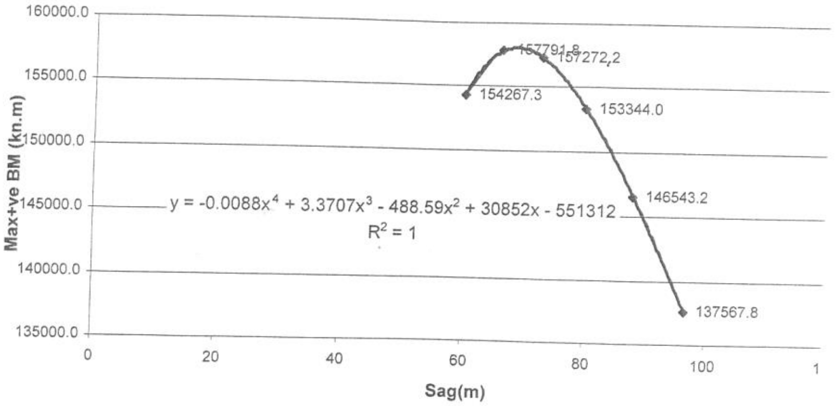


Fig (15): Max +ve BM (kN.m) vs. Cable Sag (m)

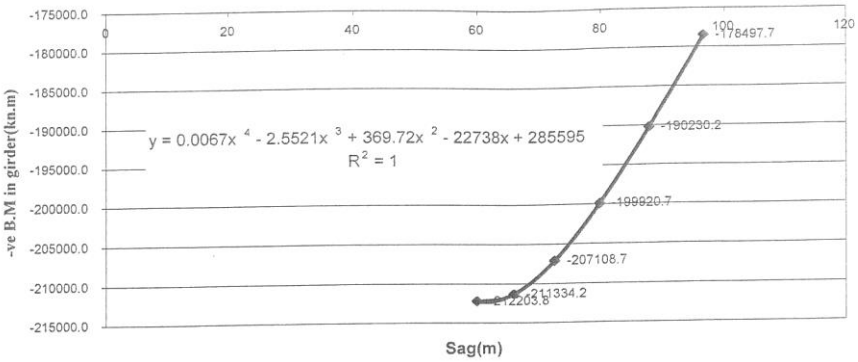


Fig (16): Max -ve BM (kN.m) vs. Cable Sag (m)

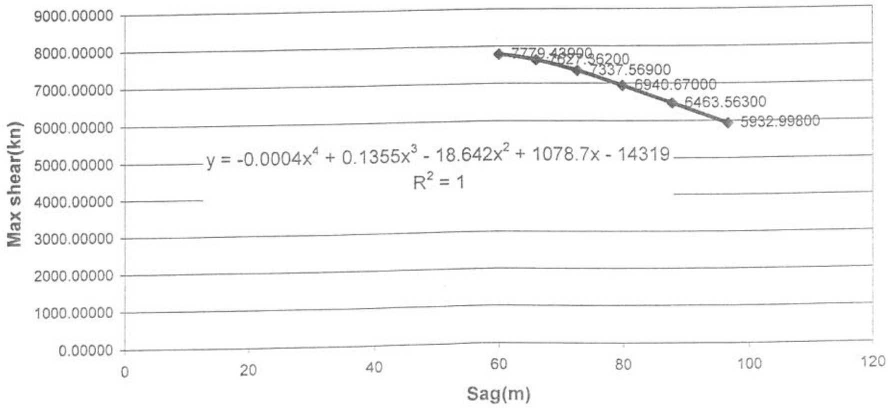


Fig (17): Max Shear (kN) vs. Cable Sag (m)

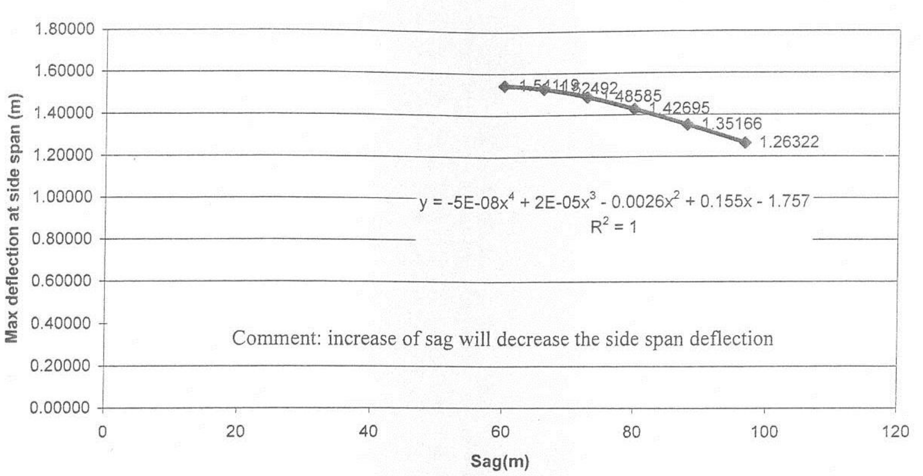


Fig (18): Max side span Deflection (m) vs. Cable Sag (m)

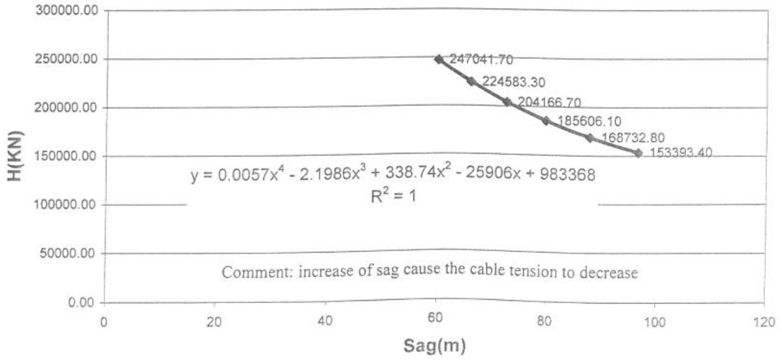


Fig (19): D.L. Horiz. Cable Tension (kN) vs. Cable Sag (m)

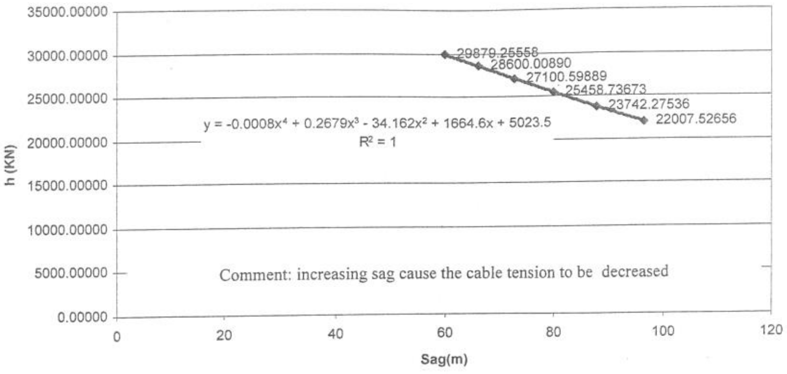


Fig (20): L.L. Horiz. Cable Tension (kN) vs. Cable Sag (m)

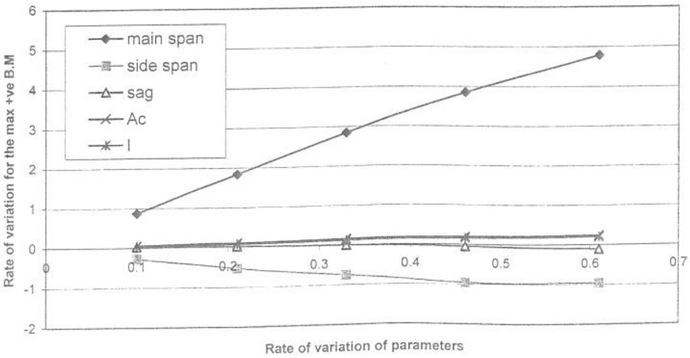


Fig (21):Max +ve BM Variation vs. Fundamental Parameters

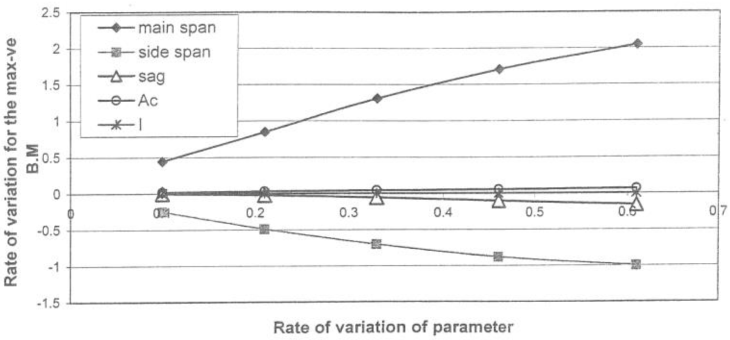


Fig (22):Max -ve BM Variation vs. Fundamental Parameters

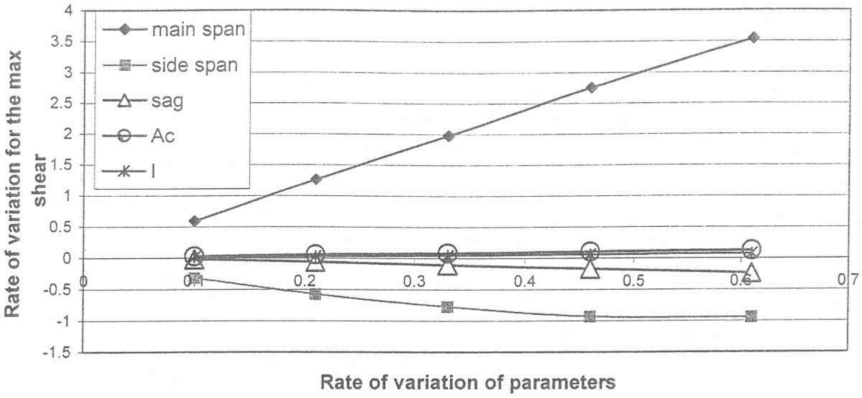


Fig (23):Max Shear Variation vs Fundamental Parameters

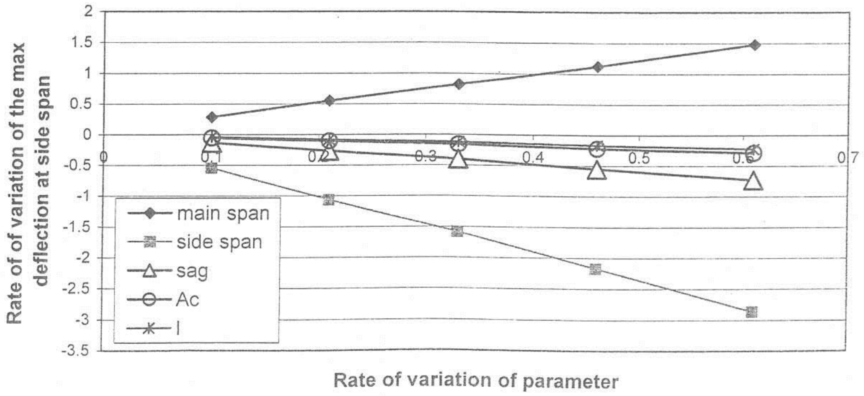


Fig (24): Max side span Deflection Variation vs. Fundamental Parameters

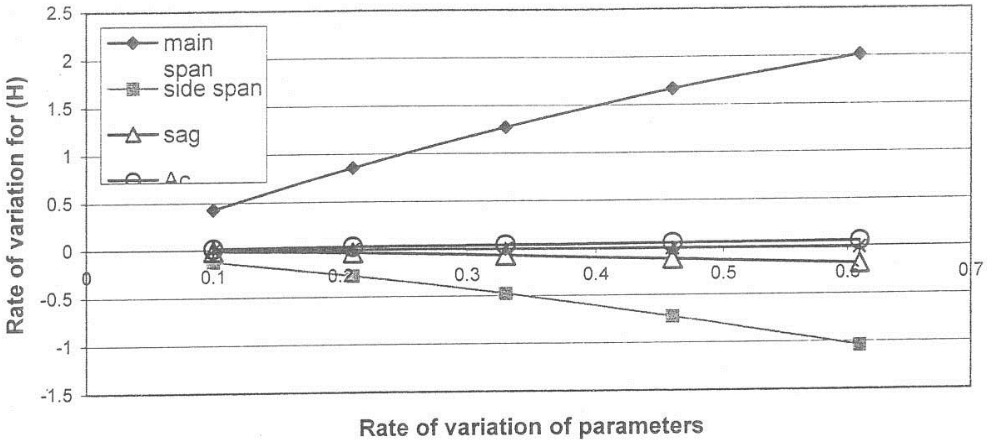


Fig (25): Initial D.L Tension Variation vs. Fundamental Parameters

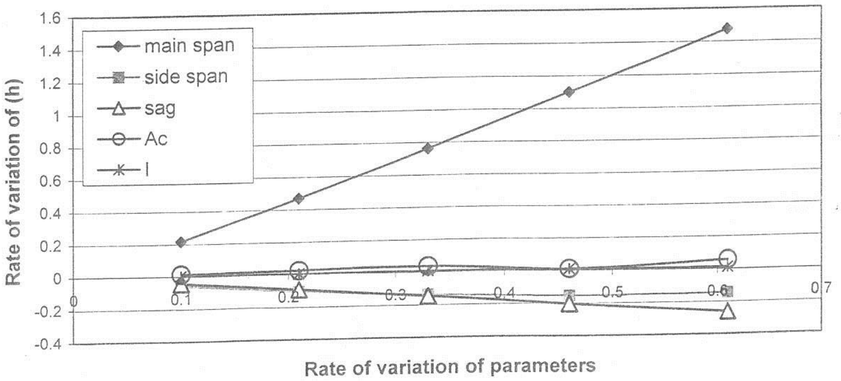


Fig (26): L.L. Induced Cable Tension Variation vs. Fundamental Parameters

b- Appendix (II)- Notations

A	*Center camber of stiffening girder at mid span……………………………..*
a_1	Center camber of stiffening girder at side span
A_c	Cable x-section
A_g	Girder x-section
BM	Bending Moment
C1Li, C2Li	Integration Constants C_1 & C_2 on left segment of span i
C1Mi, C2Mi	Integration Constants C_1& C_2 on the loaded segment of span i
C1Ri, C2Ri	Integration Constants C_1 & C_2 on the right segment of span i
d	Cable sag at mid span
d_1	Cable sag at side spans
E	Modulus of Elasticity of girder or truss

E_c	Modulus of Elasticity of the cable
g	Gravity acceleration
H	Horizontal tension in cable due to dead load
h	Horizontal tension in cable due to live loads
I , I_1	Moments of inertia of the stiffening system in mid and side spans respectively
i	Span number i=1 for left span i=2 for mid span and i=3 for right side span
$k,m,j,k_1,m_1,j_1,k_2,m_2,j_2$	Bridge spans segments
L, L_1	Length of main & side spans respectively
L_c ,L_{c1}	Lengths of cable arcs on main and side spans respectively
l, l_1	Cable length functions used in the cable equation
M	Resultant bending moment at any section of girder or truss
M_o	Simple beam bending moment due to live load
P , P_1 , P_2	Uniform applied live loads on midspan, left span and right side span respectively
P_g	Part of the live load carried by the girder
q , q_1 , q_2	Live load carried by the cable on mid, left & right spans respectively
R_{1i} , R_{2i}	Simple beam end reactions at girder supports of span I

T	Bending moment at any section, x, due to continuity
T_1 , -T_2	Bending moments in the girder at left and right supports respectively
t	Temperature change
U_1, U_2	Slope-change functions at towers
V	Total vertical shear at any section , x, of the girder or truss
v , v_1 , v_2	Deflection of the girder or truss at any section in mid, left, and right spans respectively due to live loads
v', v_1', v_2'	Slope of the deflected shape of the girder or truss at any section in mid, left, and right spans respectively
w	Uniformly distributed dead load on the cable
x , x_1 , x_2	Abscissas on mid, left and right spans respectively measured form right end of each span
y , y_1	Cable ordinates on main and side spans respectively
z , z_1	Stiffening girder ordinates on main and side spans respectively due to camber
θ, θ_1	Slope of cable chord in mid & side spans respectively
α	Coefficient of thermal expansion